Analysis on Algorithm of Wavelet Transform
And Its Realization in C Language

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Abstract—Wavelet analysis is a branch of mathematics which formed and developed in the late 1980s. With the development and maturation of wavelet theory and application, wavelet transform has been widely applied to most aspects of engineer, but usually only use the MATLAB wavelet toolbox to do program. In order to reflect the algorithm and connotation of wavelet transform deeply, we should write the program ourselves to realize some functions. In this paper, we will introduce the theory and algorithm analysis on Mallat algorithm of wavelet transform. And then, we will give the program flowchart of this algorithm which is realized in C language. Finally we will take the DB2 wavelet for example to program the wavelet transform in C language, and using MATLAB to draw waveform figures of using this algorithm and MATLAB wavelet toolbox to compare the results.

Index Terms— Wavelet transform, Mallat, DB2, C language, MATLAB

I. INTRODUCTION

Wavelet analysis is a branch of mathematics which formed and developed in the late 1980s. With the development and maturation of wavelet theory and application, currently it has applied to the signal processing, image processing and compression, data fusion, etc. Compare with Fourier transform, wavelet analysis indicate the signal features in both time domain and frequency domain, and it has good local characteristics. These are more conducive for us to analysis signals.

In 1910, Haar put forward the simplest wavelet--Haar wavelet, then Morlet firstly proposed the wavelet formula of translation and expansion, it was used in geological exploration. In 1988, Mallat proposed multi-resolution analysis theory (MRA), it unite several unrelated fields together, such as the mirror image filtering of speech recognition, subband coding and the pyramid method of image processing. In 1995, Sweldens proposed “lifting wavelet” and the concept of the second generation wavelet.

The realizations on algorithm of wavelet transform are not given in general books, only use the MATLAB wavelet toolbox to do program. In order to reflect the algorithm and connotation of wavelet transform deeply, we should write the program ourselves to realize some functions. So in this paper, we will introduce the theory and algorithm analysis on Mallat algorithm of wavelet transform. And then, we will give the program flowchart of this algorithm which is realized in C language.

II. ANALYSIS ON MALLAT ALGORITHM

A. Mallat algorithm

Mallat algorithm can realize the decomposition and reconstruction of signal which does not need to know its concrete structure and coefficient of scale functions and wavelet functions. And this algorithm can make the length of signal decomposition reduced by half each time, so it is a fast algorithm. The function figure of the two channel filter groups which is used to realize the decomposition and reconstruction of wavelet transform is shown in figure 1. You can see the specific process of the algorithm in [1].

In figure 1, ‘h’ is low-pass filter, ‘g’ is high-pass filter, ‘↑’ is up sampling, ‘↓’ is down sampling.

\[
\begin{array}{c}
\begin{array}{c}
\frac{a_{j-1}}{2} \\
\frac{g}{2} \\
\frac{2}{1} \\
\end{array} \end{array}
\begin{array}{c}
\begin{array}{c}
\frac{a_{j-1}}{2} \\
\frac{g}{2} \\
\frac{2}{1} \\
\end{array} \end{array}
\begin{array}{c}
\begin{array}{c}
\frac{h}{2} \\
\frac{g}{2} \\
\frac{2}{1} \\
\end{array} \end{array}
\begin{array}{c}
\begin{array}{c}
\frac{a_{j}}{2} \\
\frac{g}{2} \\
\frac{2}{1} \\
\end{array} \end{array}
\end{array}
\]

Figure 1. Wavelet transform : decomposition and reconstruction.

They can be transformed for:

\[
\begin{align*}
\alpha_{j,n} &= \sum_{k} h(2n-k) \alpha_{j-1,k} \\
\delta_{j,k} &= \sum_{k} g(2n-k) \alpha_{j-1,k} \\
\alpha_{j,n} &= \sum_{k} h(2n-k) \alpha_{j-1,k} \\
\delta_{j,k} &= \sum_{k} g(2n-k) \alpha_{j-1,k}
\end{align*}
\]
(2) Mallat formula of wavelet reconstruction:

\[ a_{j-1,n} = \sum_k h(n-2k) d_{j,k} + \sum_k g(n-2k) d_{j,k} \quad (4) \]

In the formulas, ‘h’ is scale function, ‘g’ is wavelet function, \( a_{j,n} \) is smooth coefficients of the j layer and \( d_{j,n} \) is detailed coefficients of the j layer. You can see the specific process in [2].

B. Analysis on algorithm

Here we take DB2 wavelet for example, the program flowcharts of its realization in C language are shown in figure 2.

In chapter 2 section A, the Mallat formula of wavelet decomposition and reconstruction has given, take DB2 wavelet for example, the formula can be transformed for:

\[
\begin{align*}
    a_{j,n} &= \sum_{m=0}^{3} h(m) a_{j-1,m+2n} \\
    d_{j,n} &= \sum_{m=0}^{3} h(m) a_{j-1,m+2n}
\end{align*}
\]

(5)

\[
\begin{align*}
    a_{j-1,n} &= 3 \sum_{k=0}^{3} h(n-2k) a_{j,k} + 3 \sum_{k=0}^{3} g(n-2k) d_{j,k} \quad (6)
\end{align*}
\]

The scale function coefficients of DB2 wavelet are:

\[
\begin{align*}
    h_0 &= \frac{1+\sqrt{3}}{4\sqrt{2}} \approx 0.48296, \quad h_1 = \frac{3+\sqrt{3}}{4\sqrt{2}} \approx 0.83651 \\
    h_2 &= \frac{3-\sqrt{3}}{4\sqrt{2}} \approx 0.22414, \\
    h_3 &= \frac{1-\sqrt{3}}{4\sqrt{2}} \approx -0.12940
\end{align*}
\]

Its wavelet function coefficients are:

\[
\begin{align*}
    g_0 &= -h_3, \quad g_1 = h_2, \quad g_2 = -h_1, \quad g_3 = h_0.
\end{align*}
\]

From the formula we can know, if a initial sequence \( \{a_{j-1}\} \) is given, the sequence \( \{a_j\} \), \( \{a_{j+1}\} \) can be computed. We can use periodic continuation at the boundary of the data.

Through the analysis above, we can compile the C program of wavelet transform as follow:

```c
#include<stdio.h>

void DB2(float signal[], int n, float result[][int m])
{
    int i, j, m;
    float *t;
    float h0, h1, h2, h3;
    float g0, g1, g2, g3;
    h0 = 0.48296; h1 = 0.83651; h2 = 0.22414; h3 = -0.12940;
    g0 = -h3; g1 = h2; g2 = -h1; g3 = h0;
    t = result;
    j = 0;
    m = n >> 1;
    for(i = 0; i < m; i++)
    {
        for(j = 0; j < m; j++)
        {
            t[i] = signal[2*i]*h0 + signal[2*i+1]*h1 + signal[2*i+2]*h2 + signal[2*i+3]*h3;
            t[i+m] = signal[2*i]*g0 + signal[2*i+1]*g1 + signal[2*i+2]*g2 + signal[2*i+3]*g3;
        }
    }
}
```

Description: it need to use MATLAB to make the load original signal be generated to a .dat files before C program call the .dat file. In C program, the results of DB2 wavelet processing need to put into another .dat file. In this way, we can use MATLAB to call the .dat file, and draw waveform figures of the algorithm.

III. REALIZATION OF MALLAT ALGORITHM IN C LANGUAGE
float signal[500], result[500];
int i,j,m;
float h0, h1, h2, h3;
float g0, g1, g2, g3;
fp=fopen("data.dat","rb");
for(i=0;i<500;i++)
    fread(&signal[i], sizeof(float), 1, fp);
fclose(fp);
for(i=0;i<500;i++)
    printf("%f", signal[i]);
DB2(signal, 500, result, 500);
fp=fopen("data1.dat","wb");
for(i=0;i<500;i++)
    fwrite(&result[i], sizeof(float), 1, fp);
fclose(fp);
ch=getchar();
}

The realization of periodic continuation at the boundary [3][4] is through \((2 * j) \mod n\) in the array.
Wavelet reconstruction is an inverse course of wavelet decomposition, its concrete realization will no longer list.
The reader who is interested in it can have a try.

IV. CONCLUSIONS

Run the C program above, and call the .dat files which is generated by this program through MATLAB. The result of DB2 wavelet decomposition is shown in figure 3. In order to form contrast, we also use the MATLAB toolbox to realize the DB2 wavelet decomposition, and its result is also shown in figure 3. From the figures, we can see that the two waveform figures are similar. This shows that the algorithm we raised is feasible, and the C program we compiled can realize the algorithm successful.

This is the program of calling .dat files and draw waveform figures through MATLAB [5]:

```c
load noissin;
s=noissin(1:500);
subplot(3,1,1)
plot(s);
title('original signal');
fid=fopen('data.dat','w');
fwrite(fid,s,'float');
fclose(fid);
subf=fopen('data1.dat','r');
[x,l]=fread(fid,'float');
subplot(3,1,2)
plot(x);
Ylabel('DB2 C');
[c,l]=wavedec(s,1,'db2');
subplot(3,1,3);
plot(c);
axis([0,500,-2,2]);
Ylabel('toolbox');
```

This paper only gives the C language realization of one-dimensional wavelet transform. And the two-dimensional wavelet transform can be realized in C language based on it. We hope this will help the readers who want to achieve wavelet transform themselves.

REFERENCES